HEAT TRANSFER IN A BED OF LUMP MATERIALS IN A COUNTERFLOW WITH HEAT SOURCES

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The process of heating or cooling of spherical lumps moving in a gas counterflow is analyzed for cases when the strength of the heat sources (sinks) is a linear function of the material temperature. By making a simple substitution, the formulas obtained can be used to calculate the heating of lump materials in a parallel gas flow.

The intensity of the heat release or heat absorption that generally takes place in shaft furnace charges is related to the rates of the physical and chemical processes and the energies of the latter.

In the presence of uniformly distributed heat sources or sinks the problem of finding the temperature field in a spherical particle and over the thickness of the bed is considerably complicated. The problem was investigated in [1, 2], but the use of these results to analyze the operation and design of industrial equipment is not recommended, since the solution is presented in a very general form inconvenient for numerical calculations. It should also be pointed out that in a number of practical cases of shaft furnace operation, the form of the expressions obtained does not permit the use of the results of calculations of the corresponding coefficients obtained for the problem of heating and cooling of spherical particles in a counterflow without heat sources [3,4].

The elimination of these shortcomings essentially involves a new solution of the problem and its analysis. It is to this task that the present article is devoted. We note that solution and analysis of the problem are also necessary to create a mathematical model of the technical processes in moving-bed equipment.

We will consider the stationary process. In this case it is necessary to find the temperature field in one of the particles as a function of time and also the gas temperature variation over the thickness of the bed in the presence of a heat source of strength q. In the general case q depends on the concentration of reactants, the reaction time, the activation energy, and other factors. However, within a certain temperature interval it is always possible to express q as a linear function of the form

$$q = q_0 + q_t(t_m - t_m'),$$

in which q_0 is the continuous source and q_t is the intensity of the source whose strength is proportional to the excess temperature. When heat is released in the treated material q_0 and $q_t\Delta t$ are positive; when heat is absorbed, they are negative.

In dimensionless form the problem is described mathematically by the heat conduction equation

$$\frac{\partial \vartheta}{\partial \mathrm{Fo}} - \mathrm{Po}' - \mathrm{Po}\,\vartheta = \frac{\partial^2 \vartheta}{\partial \rho^2} + \frac{2}{\rho}\frac{\partial \vartheta}{\partial \rho} \tag{1}$$

with boundary conditions: at the surface of the spherical particle

$$\left.\frac{\partial \vartheta}{\partial \rho}\right|_{\rho=1} = \mathrm{Bi}\,(\theta - \vartheta)|_{\rho=1}; \tag{2}$$

at the center of the particle

$$\left.\frac{\partial \vartheta}{\partial \rho}\right|_{\rho=0} = 0; \tag{3}$$

from the heat balance equation

$$\frac{\partial \theta}{\partial \mathrm{Fo}} = 3m\left.\left(\frac{\partial \vartheta}{\partial \rho}\right)\right|_{\rho=1}; \tag{4}$$

and from the initial conditions

$$\text{at } \mathrm{Fo} = 0 \quad \vartheta = 0 \text{ and } \theta = 1. \tag{5}$$

The problem is solved by an operational method. After carrying out a Laplace transformation with respect to relative time (Fo), it is possible to obtain an equation for the transform of the material temperature. This has the form

$$\bar{\vartheta} = \frac{\mathrm{Po}'}{(s-\mathrm{Po})s} + \{[\mathrm{Po}' - (s-\mathrm{Po})] \times \times \mathrm{sh}(\sqrt{s-\mathrm{Po}}\,\rho)\}\left\{\rho(s-\mathrm{Po})\left(3m - \frac{s}{\mathrm{Bi}}\right) \times \times (\sqrt{s-\mathrm{Po}}\,\mathrm{ch}\sqrt{s-\mathrm{Po}} - - \mathrm{sh}\sqrt{s-\mathrm{Po}} - s\,\mathrm{sh}\sqrt{s-\mathrm{Po}})\right\}^{-1}. \tag{6}$$

Using the decay theorem [5]

$$f(s+a) \rightarrow \exp(-a\,\mathrm{Fo})\,L^{-1}[f(s)], \tag{7}$$

we obtain the inverse transform

$$\vartheta = -\frac{\mathrm{Po}'}{\mathrm{Po}} + \left[\frac{\mathrm{Po}'}{\mathrm{Po}} + L^{-1}\left\{\left((\mathrm{Po}' - s) \times \times \mathrm{sh}(\rho\sqrt{s})\right)\left(\rho s\left[\left(3m - \frac{s+\mathrm{Po}}{\mathrm{Bi}}\right)(\sqrt{s}\,\mathrm{ch}\sqrt{s} - - \mathrm{sh}\sqrt{s}) - (s+\mathrm{Po})\,\mathrm{sh}\sqrt{s}\right]\right)^{-1}\right\}\right] \exp(\mathrm{Po}\,\mathrm{Fo}). \tag{8}$$

The expression in braces can be represented as a ratio of generalized polynomials, for which the numerator and denominator must be divided by $\sqrt{s}$.

After equating the denominator to zero, we obtain a characteristic equation for determining the roots, from which it follows that $s_0 = 0$. Therefore, instead of (8) we can write

$$\vartheta = -\frac{\mathrm{Po}'}{\mathrm{Po}} + \exp(\mathrm{Po\,Fo})\,L^{-1}\left\{\left((\mathrm{Po}' - s)\times \frac{\mathrm{sh}(\rho\sqrt{s})}{\rho s}\right)\left(\left(3m - \frac{s+\mathrm{Po}}{\mathrm{Bi}}\right)(\sqrt{s}\,\mathrm{ch}\sqrt{s} - \mathrm{sh}\sqrt{s}) - (s+\mathrm{Po})\,\mathrm{sh}\sqrt{s}\right)^{-1}\right\}. \quad (9)$$

We must now determine and investigate the roots of the equation

$$\left(3m - \frac{s+\mathrm{Po}}{\mathrm{Bi}}\right)(\sqrt{s}\,\mathrm{ch}\sqrt{s} - \mathrm{sh}\sqrt{s}) - (s+\mathrm{Po})\,\mathrm{sh}\sqrt{s} = 0. \quad (10)$$

For the approximate calculation of the first two roots of the problem, it is possible to use the formula obtained by solving the same problem by the Galerkin method [6]:

$$s_{1,2} = \frac{3\mathrm{Bi}}{6+\mathrm{Bi}}\left\{-\left[3 - 3m - \mathrm{Po}\left(\frac{1}{\mathrm{Bi}} + \frac{1}{6}\right)\right] \pm \left(\left[3 - 3m - \mathrm{Po}\left(\frac{1}{\mathrm{Bi}} + \frac{1}{6}\right)\right]^2 - 12m\,\mathrm{Po}\left(\frac{1}{\mathrm{Bi}} + \frac{1}{6}\right)\right)^{1/2}\right\}. \quad (11)$$

From this expression it follows that when Po < 0 both roots are real, the first root having a plus sign and the second a minus sign. As Po → 0, one real root vanishes, while the second, as in the preceding case, remains less than zero. In the presence of a positive heat source (Po > 0) there may be either two real roots or, if

$$\left[3 - 3m - \mathrm{Po}\left(\frac{1}{\mathrm{Bi}} + \frac{1}{6}\right)\right]^2 < 12m\,\mathrm{Po}\left(\frac{1}{\mathrm{Bi}} + \frac{1}{6}\right), \quad (12)$$

two complex conjugate roots. The latter condition is approximate. Moreover, at certain values of m, Po, and Bi, the roots may be double. Although the analysis has been based on Eq. (11), it is completely applicable to Eq. (10). We note, moreover, that all the roots of Eq. (10), starting with the third, are negative and, as the number of the roots increases, their values approach those corresponding to the problem without heat sources (Po' = Po = 0) [4].

To find the real negative roots we can write Eq. (10), assuming $s = -\mu^2$, in the form

$$\frac{1}{\mathrm{Bi}} = \frac{\mathrm{tg}\,\mu}{\mathrm{tg}\,\mu - \mu} - \frac{3m}{\mu^2 - \mathrm{Po}}, \quad (13)$$

which has a nondenumerable set of roots.

The real positive roots of the same equation must be found from the expression obtained from (10) by substituting ν^2 for s:

$$\nu\,\mathrm{cth}\,\nu - 1 = \frac{\nu^2 + \mathrm{Po}}{3m - \dfrac{\mathrm{Po}}{\mathrm{Bi}} - \dfrac{\nu^2}{\mathrm{Bi}}}. \quad (14)$$

From this equation it is clear that when Po > 0, there may be two positive roots, a necessary condition of the existence of these roots being 3m − (Po/Bi) > 0. When Po < 0 the denominator of (14), 3m − (Po/Bi) − (ν^2/Bi), will always pass through zero, which is consistent with the above analysis using (11).

In determining the real and imaginary parts of the complex root* $\sqrt{s} = \nu \pm i\mu$ Eq. (10) is separated into two equations:

$$[K_1 - \mu^2(\mathrm{Bi}-1)]\,\mathrm{th}\,\nu - \nu(K_2 + 3\mu^2) + (K_3 + \mu^2)\,\mathrm{tg}\,\mu\,\mathrm{th}\,\nu + \mu K_4\,\mathrm{tg}\,\mu = 0, \quad (15)$$

$$[K_1 - \mu^2(\mathrm{Bi}-1)]\,\mathrm{tg}\,\mu - (K_2 + 3\mu^2)\,\mathrm{tg}\,\mu\,\mathrm{th}\,\nu - \mu(K_3 + \mu^2) - \mu K_4\,\mathrm{th}\,\nu = 0. \quad (16)$$

For purposes of numerical calculations, Eqs. (15) and (16) may be conveniently rewritten in the form

$$\begin{aligned}&\mu^6\,\mathrm{th}\,\nu + \mu^4\{[\mathrm{th}\,\nu(\mathrm{Bi}-1) + 3\nu](\mathrm{Bi} - 1 + 3\nu\,\mathrm{th}\,\nu) + \\ &+ \mathrm{th}\,\nu(K_3 + K_4\,\mathrm{th}\,\nu) + (K_4 + K_3\,\mathrm{th}\,\nu)\} - \\ &- \mu^2\{[\mathrm{th}\,\nu(\mathrm{Bi}-1) + 3\nu](K_1 - K_2\nu\,\mathrm{th}\,\nu) - \\ &- (K_4 + K_3\,\mathrm{th}\,\nu)(K_3 + K_4\,\mathrm{th}\,\nu) + \\ &+ (K_1\,\mathrm{th}\,\nu - K_2\nu)(\mathrm{Bi} - 1 - 3\nu\,\mathrm{th}\,\nu)\} + \\ &+ (K_1\,\mathrm{th}\,\nu - K_2\nu)(K_1 - K_2\nu\,\mathrm{th}\,\nu) = 0 \end{aligned} \quad (17)$$

and

$$2\mathrm{tg}\,\mu - \frac{\nu(K_2\mathrm{Bi} + 3\mu^2) - [K_1\mathrm{Bi} - \mu^2(\mathrm{Bi}-1)]\,\mathrm{th}\,\nu}{\mu(K_3\mathrm{Bi} + \mu^2)\,\mathrm{th}\,\nu + \mu K_4\mathrm{Bi}} - \frac{\mu(K_3\mathrm{Bi} + \mu^2) + \mu K_4\mathrm{Bi}\,\mathrm{th}\,\nu}{K_1\mathrm{Bi} - \mu^2(\mathrm{Bi}-1) - \nu(K_2\mathrm{Bi} + 3\mu^2)\,\mathrm{th}\,\nu} = 0, \quad (18)$$

where

$$K_1 = 3m\,\mathrm{Bi} + \mathrm{Po}(\mathrm{Bi}-1) + \nu^2(\mathrm{Bi}-1);$$
$$K_2 = 3m\,\mathrm{Bi} - \mathrm{Po} - \nu^2;$$
$$K_3 = 3m\,\mathrm{Bi} - \mathrm{Po} - 3\nu^2;\quad K_4 = 2\nu(1-\mathrm{Bi}). \quad (19)$$

It is easy to show that there are no complex roots with a large real part. In fact, at large ν (ν > 4.0) th $\nu \approx$ cth $\nu \approx 1.0$, which after transformations gives $\mathrm{tg}^2\mu = -1$, which, of course, is not possible.

The joint solution of Eqs. (17) and (18) makes it possible to obtain the values of ν and μ. The numerical calculations should be carried out in the following order: first, expression (11) is used to determine the real and imaginary parts of the complex conjugate root.

*If the complex conjugate roots of expression (11) are denoted by $s = \delta \pm i\sigma$, then the following relations will hold among δ, σ, ν, μ:

$$\delta - \mathrm{Po} = \nu^2 - \mu^2;\quad \sigma = 2\nu\mu.$$

This makes possible a rough estimate of ν and μ using the relations presented above. The corresponding μ are determined from (17) by assigning several values of ν close to the approximate value. Then, by substituting the found ν-μ pairs into Eq. (18), one finds (graphically, numerically) those ν and μ that make the left side of the equation vanish. It should be noted that the complex roots calculated on the basis of (11) give somewhat exaggerated values of ν and μ.

The transition from the transform (7) to the inverse transform can be accomplished by means of the expansion theorem. Since the final form of the function ϑ depends on the type of roots of the characteristic equation, the relations presented below are classified according to this principle.

a. **Roots real, negative.** This case corresponds to Po > 0 and approximately $[3 - 3m - \text{Po}(1/\text{Bi} + 1/6)]^2 > 12m\,\text{Po}(1/\text{Bi} + 1/6)$. After transformations, taking $s = -\mu^2$, we finally obtain for the material temperature

$$\vartheta = -\frac{\text{Po}'}{\text{Po}} + \exp(\text{Po Fo}) \sum_{n=1}^{\infty} \left(\frac{\text{Po}'}{\mu_n^2} + 1\right) \times C_n \frac{\sin \mu_n \rho}{\rho} \exp(-\mu_n^2 \text{Fo}), \tag{20}$$

where

$$C_n = \left\{\left(\frac{\mu_n}{\text{Bi}} - \frac{\text{Po} - \mu_n^2}{2\mu_n}\right) \cos \mu_n - \left[\frac{1}{\text{Bi}}\left(1 - \frac{\text{Po} - \mu_n^2}{2}\right) + \frac{3}{2} m - 1\right] \sin \mu_n\right\}^{-1}.$$

The gas temperature can be found using (4) and (20):

$$\theta = 1 - 3m \sum_{n=1}^{\infty} \frac{\text{Po}' + \mu_n^2}{\text{Po} - \mu_n^2} \times C_n \Phi_n \{\exp[(\text{Po} - \mu_n^2)\text{Fo}] - 1\}, \tag{21}$$

where

$$\Phi_n = -\frac{\mu_n \cos \mu_n - \sin \mu_n}{\mu_n^2}.$$

Going over to the case when the strength of the continuous source is equal to zero (Po' = 0) does not present any difficulties. From the previous analysis it follows that when Po → 0 and Po' ≠ 0 one of the roots approaches zero. This makes it possible to substitute series for the trigonometric functions in (13). Finally, we obtain

$$-\text{Po} + s\left[m - 1 - \frac{\text{Po}}{3}\left(\frac{1}{\text{Bi}} + \frac{1}{2}\right)\right] + s^2\left[\left(3m - \frac{\text{Po}}{\text{Bi}}\right)\frac{4}{5!} - \frac{2}{3!\,\text{Bi}} - \frac{1}{3!} - \frac{\text{Po}}{5!}\right] + s^3\left[\left(3m - \frac{\text{Po}}{\text{Bi}}\right)\frac{6}{7!} - \frac{4}{5!\,\text{Bi}} - \frac{1}{5!} - \frac{\text{Po}}{7!}\right] + \ldots + s^n\left[\left(3m - \frac{\text{Po}}{\text{Bi}}\right)\frac{2n}{(2n+1)!} - \frac{2n-2}{(2n-1)!\,\text{Bi}} - \frac{1}{(2n-1)!} - \frac{\text{Po}}{(2n+1)!}\right] + \ldots = 0. \tag{22}$$

In view of the small values of s, it is sufficient to confine ourselves to the second power only; this gives

$$s^2\left(\frac{m}{10} - \frac{1}{\text{Bi}} - \frac{1}{6} - \frac{\text{Po}}{30\text{Bi}} - \frac{\text{Po}}{5!}\right) + s\left[m - 1 - \frac{\text{Po}}{3}\left(\frac{1}{\text{Bi}} + \frac{1}{2}\right)\right] - \text{Po} = 0. \tag{23}$$

The solution of this equation for small Po can be represented in the form

$$s = -\frac{d}{k}\left(1 + \frac{d}{k^2}\right) = -\mu^2, \tag{24}$$

where

$$\left.\begin{aligned} d &= \frac{-\text{Po}}{\frac{m}{10} - \frac{1}{3}\left(\frac{1}{\text{Bi}} + \frac{1}{2}\right) - \text{Po}\left(\frac{1}{30\text{Bi}} + \frac{1}{5!}\right)} \\ k &= \frac{m - 1 - \frac{\text{Po}}{3}\left(\frac{1}{\text{Bi}} + \frac{1}{2}\right)}{\frac{m}{10} - \frac{1}{3}\left(\frac{1}{\text{Bi}} + \frac{1}{2}\right) - \text{Po}\left(\frac{1}{30\text{Bi}} + \frac{1}{5!}\right)} \end{aligned}\right\}. \tag{25}$$

As $\mu_n \to 0$ (Po → 0) one of the terms of expression (20) can be transformed to

$$\Sigma_0 = \left[(\text{Po}' + \mu^2) + \text{Po}'(\text{Po} - \mu^2)\text{Fo} - 0.1666\mu^2\rho^2\text{Po}' - \frac{\text{Po}'}{\text{Po}} \times (\mu^4 a + \mu^2 b - 0.5\text{Po})\right][\mu^4 a + \mu^2 b - 0.5\text{Po}]^{-1}, \tag{26}$$

where

$$a = 2.5\left[\frac{m}{10} - \frac{1}{3}\left(\frac{1}{\text{Bi}} + \frac{1}{2}\right) - \text{Po}\left(\frac{1}{30\text{Bi}} + \frac{1}{5!}\right)\right];$$

$$b = \text{Po}\left(\frac{1}{2\text{Bi}} + 0.25\right) - 1.5(m - 1).$$

Substituting root (24) into (26) together with d and k from (25), and letting Po approach zero, gives for the particle temperature

$$\vartheta = -\frac{1}{m-1} - \text{Po}'\frac{3m\,\text{Bi} - 5\text{Bi} - 10}{30(m-1)^2\,\text{Bi}} + \frac{\text{Po}'\rho^2}{6(m-1)} + \frac{m\,\text{Po}'\,\text{Fo}}{m-1} + \sum_{n=1}^{\infty}\left(\frac{\text{Po}'}{\mu_n^2} + 1\right) \times (C_n)_{\text{Po}=0}\frac{\sin \mu_n \rho}{\rho} \exp(-\mu_n^2 \text{Fo}), \tag{27}$$

where μ_n are the roots of the characteristic equation

$$\frac{1}{\text{Bi}} = \frac{\text{tg}\,\mu_n}{\text{tg}\,\mu_n - \mu_n} - \frac{3m}{\mu_n^2}. \tag{28}$$

In the same way it is possible to obtain an expression for the temperature of the gas flow θ at Po = 0. However, to calculate θ at Po' ≠ 0 and Po = 0, it is better to use Eq. (4). Then

$$\theta = 1 + \frac{m\,\text{Po}'\,\text{Fo}}{m-1} - 3m \times \sum_{n=1}^{\infty} \left(\frac{\text{Po}'}{\mu_n^2} + 1\right) (C_n)_{\text{Po}=0}\, \Phi_n [1 - \exp(-\mu_n^2\,\text{Fo})]. \quad (29)$$

When Po' = 0, expressions (27) and (29) go over to the known solutions [3, 4, 7–9]. Moreover, the first three roots μ_n of Eq. (28) are presented in [3], together with the coefficients Φ_n and $(C_n)_{\text{Po}=0}$ of the first three terms of the power series (27) and (29) for calculating the temperatures at the surface and center of the sphere and the mass-averaged temperature. In [3], moveover, numerical values of μ_n, Φ_n, and $(C_n)_{\text{Po}=0}$ are presented for 14 ratios of flow specific heats m from 0.1 to 10.0 and 31 values of Bi from 0.02 to infinity.

b. **First two roots positive, different.** These cases occur when Po > 0 and approximately Po(1/Bi + 1/6) > > 3(1 − m). For the indicated cases s = ν^2. Then

$$\vartheta = -\frac{\text{Po}'}{\text{Po}} + \left[\sum_{n=1}^{2} \left(\frac{\text{Po}'}{\nu_n^2} - 1\right) C_\nu \frac{\text{sh}\,\nu_n\rho}{\rho} \times \exp(\nu_n^2\,\text{Fo}) + \sum_{n=3}^{\infty} \left(\frac{\text{Po}'}{\mu_n^2} + 1\right) C_n \frac{\sin \mu_n\rho}{\rho} \times \exp(-\mu_n^2\,\text{Fo})\right] \exp(\text{Po}\,\text{Fo}); \quad (30)$$

$$\theta = 1 - 3m \left[\sum_{n=1}^{2} \frac{\text{Po}' - \nu_n^2}{\text{Po} + \nu_n^2} \times C_\nu \Phi_\nu \{\exp[(\text{Po} + \nu_n^2)\,\text{Fo}] - 1\} + \sum_{n=3}^{\infty} \frac{\text{Po}' + \mu_n^2}{\text{Po} - \mu_n^2} C_n \Phi_n \{\exp[(\text{Po} - \mu_n^2)\,\text{Fo}] - 1\}\right], \quad (31)$$

where ν_n are the roots of the characteristic equation (14) and μ_n, those of (13),

$$C_\nu = \left\{\left(\frac{\text{Po} + \nu_n^2}{2\nu_n} + \frac{\nu_n}{\text{Bi}}\right) \text{ch}\,\nu_n + \left[\frac{1}{\text{Bi}}\left(1 - \frac{\text{Po} + \nu_n^2}{2}\right) + \frac{3}{2}\, m - 1\right] \text{sh}\,\nu_n\right\}^{-1};$$

$$\Phi_\nu = \frac{\text{sh}\,\nu_n - \nu_n\,\text{ch}\,\nu_n}{\nu_n^2}.$$

When Po < 0 only the first root can be positive. Therefore, in expressions (30) and (31) there will be only one term with a positive root, and the infinite sum must be taken from 2 to ∞.

As Po tends to zero, the root that is smaller in absolute magnitude will also approach zero, while the other root remains positive. In this case the passage to the limit is the same as above. In particular, for Po = 0 and m > 1.0 the material temperature is described by the equation

$$\vartheta = -\frac{1}{m-1} - \text{Po}'\,\frac{3m\,\text{Bi} - 5\text{Bi} - 10}{30(m-1)^2\,\text{Bi}} + \frac{\text{Po}'\rho^2}{6(m-1)} + \frac{m\,\text{Po}'\,\text{Fo}}{m-1} + \left(\frac{\text{Po}'}{\nu^2} - 1\right) \times (C_\nu)_{\text{Po}=0} \frac{\text{sh}\,\nu\rho}{\rho} \exp(\nu^2\,\text{Fo}) + \sum_{n=2}^{\infty} \left(\frac{\text{Po}'}{\mu_n^2} + 1\right) \times (C_n)_{\text{Po}=0} \frac{\sin \mu_n\rho}{\rho} \exp(-\mu_n^2\,\text{Fo}), \quad (32)$$

and the gas temperature by the equation

$$\theta = 1 + \frac{m\,\text{Po}'\,\text{Fo}}{m-1} - 3m \times \left\{\left(\frac{\text{Po}'}{\nu^2} - 1\right)(C_\nu)_{\text{Po}=0}\,\Phi_\nu [\exp(\nu^2\,\text{Fo}) - 1] - \sum_{n=2}^{\infty} \left(\frac{\text{Po}'}{\mu_n^2} + 1\right)(C_n)_{\text{Po}=0}\,\Phi_n [\exp(-\mu_n^2\,\text{Fo}) - 1]\right\}. \quad (33)$$

Values of ν, μ_n, $(C_\nu)_{\text{Po}=0}$, $(C_n)_{\text{Po}=0}$, Φ_ν, Φ_n, required to calculate the temperature fields from Eqs. (32) and (33) are presented in [3].

c. **Roots complex.** Roots of the type $\sqrt{s} = \nu \pm i\mu$ appear when Po > 0 and inequality (12) holds. The temperature of the heated particles is found from the expression

$$\vartheta = -\frac{\text{Po}'}{\text{Po}} + \frac{2\exp(A\,\text{Fo})}{M^2 + N^2} \left(\frac{UM + DN}{\rho} \times \cos 2\mu\nu\,\text{Fo} - \frac{DM - UN}{\rho} \sin 2\mu\nu\,\text{Fo}\right) + \sum_{n=3}^{\infty} \left(\frac{\text{Po}'}{\mu_n^2} + 1\right) C_n \frac{\sin \mu_n\rho}{\rho} \exp[(\text{Po} - \mu_n^2)\,\text{Fo}]. \quad (34)$$

The quantity θ must be determined from Eq. (4). Thus,

$$\theta = 1 + \frac{6mA(QM + EN) + 2\mu\nu(EM - QN)}{(M^2 + N^2)(A^2 + 4\mu^2\nu^2)} \times \left\{\exp(A\,\text{Fo})\left[\cos 2\mu\nu\,\text{Fo} - \frac{A(EM - QN) - 2\mu\nu(QM + EN)}{A(QM + EN) + 2\mu\nu(EM - QN)} \sin 2\mu\nu\,\text{Fo}\right] - 1\right\} - 3m \sum_{n=3}^{\infty} \frac{\text{Po}' + \mu_n^2}{\text{Po} - \mu_n^2} \times C_n \Phi_n \{\exp[(\text{Po} - \mu_n^2)\,\text{Fo}] - 1\}. \quad (35)$$

Here,

$$A = \text{Po} + \nu^2 - \mu^2; \quad M = (\nu^2 - \mu^2) M_1 - 2\mu\nu N_1;$$
$$N = (\nu^2 - \mu^2) N_1 + 2\mu\nu M_1;$$

$$M_1 = \frac{1}{\text{Bi}}\left[(1 - \text{Bi} - 0.5A + 1.5m\,\text{Bi})\,\text{sh}\,\nu \cos\mu + \mu\nu\,\text{ch}\,\nu \sin\mu - \left(1 + \frac{0.5\text{Po}\,\text{Bi}}{\nu^2 + \mu^2} + 0.5\,\text{Bi}\right)\nu\,\text{ch}\,\nu \times\right.$$

$$\times \cos\mu + \left(1 - \frac{0.5\,\mathrm{Po\,Bi}}{\nu^2+\mu^2} + 0.5\,\mathrm{Bi}\right)\mu\,\mathrm{sh}\,\nu\sin\mu\Big];$$

$$N_1 = \frac{1}{\mathrm{Bi}}\Big[(1 - \mathrm{Bi} - 0.5A +$$

$$+ 1.5m\,\mathrm{Bi})\,\mathrm{ch}\,\nu\sin\mu - \mu\nu\,\mathrm{sh}\,\nu\cos\mu -$$

$$-\left(1 + \frac{0.5\,\mathrm{Po\,Bi}}{\nu^2+\mu^2} + 0.5\,\mathrm{Bi}\right)\nu\,\mathrm{sh}\,\nu\sin\mu -$$

$$-\left(1 - \frac{0.5\,\mathrm{Po\,Bi}}{\nu^2+\mu^2} + 0.5\,\mathrm{Bi}\right)\mu\,\mathrm{ch}\,\nu\cos\mu\Big];$$

$$U = [\mathrm{Po}' - (\nu^2 - \mu^2)]\,\mathrm{sh}\,\nu\rho\cos\mu\rho + 2\mu\nu\,\mathrm{ch}\,\nu\rho\sin\mu\rho;$$

$$D = [\mathrm{Po}' - (\nu^2 - \mu^2)]\,\mathrm{ch}\,\nu\rho\sin\mu\rho + 2\mu\nu\,\mathrm{sh}\,\nu\rho\cos\mu\rho;$$

$$Q = [\mathrm{Po}' - (\nu^2 - \mu^2)]\,Q_1 + 2\mu\nu E_1;$$

$$E = [\mathrm{Po}' - (\nu^2 - \mu^2)]\,E_1 - 2\mu\nu Q_1;$$

$$Q_1 = \nu\,\mathrm{sh}\,\nu\sin\mu + \mu\,\mathrm{ch}\,\nu\cos\mu - \mathrm{ch}\,\nu\sin\mu;$$

$$E_1 = \nu\,\mathrm{ch}\,\nu\cos\mu - \mu\,\mathrm{sh}\,\nu\sin\mu - \mathrm{sh}\,\nu\cos\mu.$$

The coefficients ν and μ are roots of Eqs. (15) and (16). The existence of complex roots and the form of Eqs. (34) and (35) indicate the wave character of the temperature variations of the particles and the gas over the thickness of the bed.

d. **Root double.** Analysis shows that the double root may be both positive and negative. In this case in order to determine the inverse transform it is convenient to use the expansion theorem [5] in the form

$$f_2(\mathrm{Fo}) = \lim_{s\to s_m} \frac{d}{ds}\,\frac{\Phi(s)(s - s_m)^2}{\psi(s_m)}\exp(s\,\mathrm{Fo}). \qquad (36)$$

If $s_m = \nu^2$ the temperature field of the particle is described by the following equation:

$$\vartheta = -\frac{\mathrm{Po}'}{\mathrm{Po}} + \frac{2(\mathrm{Po}' - \nu^2)}{\psi''(\nu)}\left(\frac{\rho\,\mathrm{cth}\,\nu\rho}{2\nu} - \right.$$

$$\left. - \frac{\psi'''(\nu)}{3\psi''(\nu)} - \frac{1}{\mathrm{Po}' - \nu^2} + \mathrm{Fo}\right)\frac{\mathrm{sh}\,\nu\rho}{\rho} \times$$

$$\times \exp[(\mathrm{Po} + \nu^2)\,\mathrm{Fo}] + \sum_{n=3}^{\infty}\left(\frac{\mathrm{Po}'}{\mu_n^2} + 1\right)\times$$

$$\times C_n\frac{\sin\mu_n\rho}{\rho}\exp[(\mathrm{Po} - \mu_n^2)\,\mathrm{Fo}], \qquad (37)$$

and the temperature of the gas flow by the equation

$$\theta = 1 + 3m\,\frac{2(\mathrm{Po}' - \nu^2)\,\nu^2\Phi_\nu}{\psi''(\nu)(\mathrm{Po} + \nu^2)}\times$$

$$\times\left[\left(\frac{\mathrm{sh}\,\nu}{2\nu^2\Phi_\nu} + \frac{\psi'''(\nu)}{3\psi''(\nu)} + \frac{1}{\mathrm{Po}' - \nu^2} + \frac{1}{\mathrm{Po} + \nu^2}\right)\times\right.$$

$$\left.\times\{\exp[(\mathrm{Po} + \nu^2)\,\mathrm{Fo}] - 1\} - \mathrm{Fo}\exp[(\mathrm{Po} + \nu^2)\,\mathrm{Fo}]\right] -$$

$$-3m\sum_{n=3}^{\infty}\frac{\mathrm{Po}' + \mu_n^2}{\mathrm{Po} - \mu_n^2}\,C_n\Phi_n(\exp[(\mathrm{Po} - \mu_n^2)\,\mathrm{Fo}] - 1). \qquad (38)$$

In these expressions

$$\psi''(\nu) = \frac{1}{\mathrm{Bi}}\,[1 - \mathrm{Bi} - 0.5\mathrm{Po} - 0.25\mathrm{Po\,Bi} +$$

$$+ 1.5m\,\mathrm{Bi} - \nu^2(1.5 + 0.25\mathrm{Bi})]\,\mathrm{sh}\,\nu +$$

$$+ \frac{1}{\mathrm{Bi}}\Big[\nu(0.75m\,\mathrm{Bi} - 0.5 - 1.25\mathrm{Bi} - 0.25\mathrm{Po}) -$$

$$-0.25\,\frac{\mathrm{Po\,Bi}}{\nu} - 0.25\nu^3\Big]\,\mathrm{ch}\,\nu;$$

$$\psi'''(\nu) = \frac{1}{\mathrm{Bi}}\,[0.5(0.75m\,\mathrm{Bi} - 0.5 - 1.25\mathrm{Bi} -$$

$$-0.25\mathrm{Po}) - 0.125\,\frac{\mathrm{PoBi}}{\nu^2} - 0.125\nu^2 - 1.5 -$$

$$-0.25\mathrm{Bi}]\,\mathrm{sh}\,\nu + \frac{1}{\mathrm{Bi}}\Big[0.5(0.75m\,\mathrm{Bi} - 0.25\mathrm{Po\,Bi} +$$

$$+ 0.5 - 2.25\mathrm{Bi} - 1.5\mathrm{Po})\frac{1}{\nu} - 0.125\,\frac{\mathrm{Po\,Bi}}{\nu^3} -$$

$$-0.125\nu\,\mathrm{Bi} - 1.125\nu + 0.75\,\frac{m\,\mathrm{Bi}}{\nu}\Big]\,\mathrm{ch}\,\nu.$$

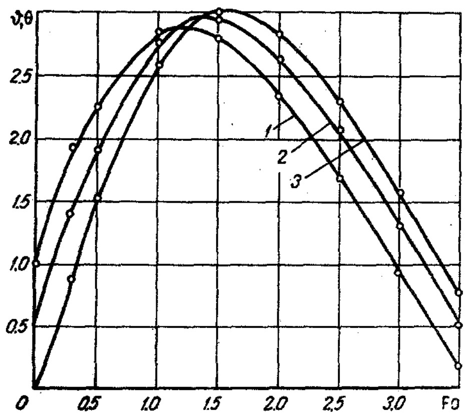
If $s_m = -\mu^2$, then in Eqs. (37) and (38) it is necessary to substitute $i\mu$ for ν and go over from hyperbolic to trigonometric functions, in accordance with the usual rules.

The application of the equations obtained is illustrated by the results of calculations of the temperature distribution over the thickness of the bed for the following conditions: m = 0.8; Bi = 2.0; and Po' = Po = = 0.15.

In this case calculation of inequality (12) points to the existence of complex conjugate roots, whose value, in accordance with (11), is approximately equal to $\nu_{app} = 0.3851$ and $\mu_{app} = 0.8205$. A more accurate determination of ν and μ from the transcendental equations (15) and (16) finally gives $\nu = 0.38480$ and $\mu = 0.8068$. The subsequent roots are $\mu_3 = 7.9619$ and $\mu_4 = 11.0785$.

The variation of the gas temperature and the temperatures at the surface and center of the particle are shown in the figure. As follows from the figure, the process can be divided into two periods: heating and cooling. Each has its own specific temperature distribution over the cross section of the particle, created by the action of the heat sources. In the first period the charge is rapidly heated and toward the end of this period heat transfer between the charge and the gas slows down. At time Fo = 1.109, which is found from the joint solution of Eqs. (25) and (24) for $\rho = 1.0$, the temperature of the gas and the surface temperature of the particle are equal. Somewhat later (Fo = 1.322) the temperatures at the center and at the surface of the particle become the same. This moment is established by solving the equations obtained from (34) with $\rho = 0$ and $\rho = 1.0$. During this time the temperature field of the particle undergoes a deformation that ends $\Delta\mathrm{Fo} \approx 1.0$ from the moment of temperature equalization.

The case in question is characterized by damping of the temperature oscillations, since the exponent of the exponential function $\mathrm{Po} + \nu^2 - \mu^2$ is negative. In the presence of high-strength heat sources continuous oscillations may appear. The nature of the temperature field in the bed (figure) indicates that temperatures higher than those on the inlet and outlet sections may develop in the apparatus.



Temperature variation over the thickness of the bed: 1) gas; 2) surface; 3) center of the particle.

The equations describing the distribution of particle and gas temperatures over the thickness of the bed will also be valid for calculating the heating of spherical particles in a parallel flow, if the ratio m of flow specific heats (water equivalents) is replaced by minus m.

NOTATION

t is the temperature; ρ is the relative coordinate; R is the particle radius; α is the heat-transfer coefficient; λ is the thermal conductivity; a is the thermal diffusivity; τ is the time from the moment the particles are loaded into the bed; H is the thickness of the bed; w is the velocity of the particles in the bed; $\vartheta = (t_m - t'_m)/(t''_g - t'_m)$; $\theta = (t_g - t'_m)/(t''_g - t'_m)$ are the temperature criteria for the material and the gas flows; $Bi = \alpha R/\lambda_m$ is the Biot number; $Fo = a\tau/R^2 = aH/w_m R^2$ is the Fourier number; $Po' = -q_0 R^2/\lambda_m (t''_g - t'_m)$; $Po = q_t R^2/\lambda_m$ is the Pomerantsev number; q_0 is the strength of the continuous source; q_t is the source whose strength is proportional to the excess temperature. The subscripts m and g indicate that the quantity in question relates to the material or the gas; single and double primes denote parameters characterizing the state of the medium on the inlet and outlet sides, respectively.

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